Unfolding regularities between population and irrigated agriculture might increase our capacity to predict their coevolution and better ensure food security and environmental welfare. Here I use three different data sets with detailed information at the national level for ~70% of the countries of Africa, Asia, the Americas, and Europe between 1950 and 2017 to show that irrigated areas might grow disproportionally for a given increase in population, e.g., with $\beta > 1$. The results are robust across continents, time series, population cut-offs, and variations in the area accounted for irrigation by official institutions and independent scholars. This systematic pattern suggests the existence of an underlying law driving the growth rate of irrigated areas that transcends local particularities and can be well approximated by a power function of population, specially in the case of the Americas, Asia, and Europe. Non-linearities derived from the open-ended growth rate of irrigated areas should be taken into consideration when designing irrigation policies in order to avoid unexpected environmental costs.

**Key words:** allometry; intensive agriculture; irrigation; scaling; standard major axis; statistics; sustainability.

**INTRODUCTION**

How does the extension of irrigated agriculture change with a change in population size? This question is at the heart of sustainable agrarian development and food security policies. We know that larger populations and higher population densities are usually linked to larger irrigated areas (Boserup 1965, 1981), and that processes of extension and reduction of irrigated areas correlate very often with demographic bursts and drops (Puy et al. 2017). Enlargements of the area under irrigated areas have also been widely documented in historical contexts in connection with population growth linked to the arrival of new settlers, e.g., during the feudal conquest of al-Andalus (Iberian Peninsula, 11th–15th centuries AD; Torró 2006, Guinot and Esquilache 2012). The link between changes in population size and irrigated areas does not necessarily reflect a simple cause–effect relationship (Morrison 1994), but a more nuanced process in which both variables influence each other through different feedback loops (Puy et al. 2017). Unfolding whether these dynamics follow consistent rules regardless of the scale or geographical focus under consideration, particularly on the extent to which irrigated areas increase for a given increase in population, might allow us to better understand their coevolution. This has implications for achieving a more sustainable balance between human and environmental welfare, especially considering that population is expected to reach approximately 10 billion by 2050 and irrigated agriculture is already a major source of freshwater consumption and environmental stress (UN 2012, 2015).

Here I assess the relation rate between irrigated areas and population using large-scale data sets with detailed information for ~70% of the countries of Africa, Americas, Asia, and Europe between 1950 and 2017. I aim at providing insights into the hidden mechanisms that drive their coevolution by unraveling patterns that transcend local particularities and trajectories. I rely on *scaling*, a conceptual framework that allows to study how one variable of a system scales with another variable of the same system or with the system as a whole (West 2017). With this approach, I show that irrigated areas and population behave in an overall consistent way following a nonlinear relationship that has important implications for our management of water and land resources. The study also exemplifies the potential of integrating scaling to study how food-producing systems change their properties when they change their size, a topic directly related to the long-standing debate on whether “small” or “large” agrarian areas are more “beautiful” (Schumacher 1973, Adams 1990, Dillon 2011).

**MATERIALS AND METHODS**

The conceptual framework

Scaling provides a suitable framework to describe how irrigated areas and population coevolve. Irrigated areas scale with population if their relationship can be adequately modeled as

$$ Y_{i,t} = \alpha X_{i,t}^\beta $$  \hspace{1cm} (1)

where $X$ and $Y$, respectively, reflect population and irrigated area for sample $i$ at time $t$, $\alpha$ is a constant and $\beta$ the scaling
exponent (or an elasticity, as in economics; Lobo et al. 2013). For a relative increase in population, \( \Delta Y/X \), the relative increase in irrigated area is then given by \( \Delta Y/Y = (\beta - 1)\Delta Y/X \) (Bettencourt et al. 2010). If \( \beta > 1 \), \( \Delta Y/Y \approx 0 \), and the growth rate between population and irrigated area is proportional or linear on average. If \( \beta > 1 \) (or \( \beta < 1 \)), \( \Delta Y/Y > 0 \) (or \( \Delta Y/Y < 0 \)) and the irrigated area grows faster/superlinearly (or slower/sublinearly) for every relative increase in population. The ratio \( \Delta Y/Y \) does not depend on \( X \), but on \( \beta \), a property known as scale-invariance. For a given change in population, therefore, the extent of the irrigated area changes by the same value of \( \beta \) regardless of the initial population size. Scaling has been applied to understand the growth rate of animals and their metabolic rates (White et al. 2007) or of cities and attributes such as wealth creation capacity, pace of innovation, CO\(_2\) emissions, or population density (White et al. 2007. Bettencourt 2013, Fraggias et al. 2013, Orman et al. 2014, Cesaretti et al. 2016). To my knowledge, and despite its potential to unfold universal patterns between agrarian areas and population, this approach has not been integrated in the study of the dynamics of food-producing systems as yet.

The data sets

I analyze how irrigated areas scale with population using three different custom-built data sets. This aims at assessing the robustness of \( \beta \) against variations in the extension accounted for irrigation. It is known that agrarian areas reported by official institutions via national surveys and/or governmental publications differ substantially from those appraised by independent scholars through field surveys, remote sensing, Google Earth, or historical imagery (Young 1999). In the case of irrigation agriculture, the discrepancies are large: the irrigated areas attested by Thenkabail et al. (1999) are large: the irrigated areas attested by Thenkabail et al. (2009) are large: the irrigated areas attested by Thenkabail et al. (2009). In the case of irrigation agriculture, the discrepancies are large: the irrigated areas attested by Thenkabail et al. (2009) are large: the irrigated areas attested by Thenkabail et al. (2009) are large: the irrigated areas attested by Thenkabail et al. (2009) are large: the irrigated areas attested by Thenkabail et al. (2009) are large: the irrigated areas attested by Thenkabail et al. (2009).

The first data set is based on FAO’s Aquastat values on the total Area Equipped for Irrigation (AEI) at the national level between 1958 and 2017 (FAO 2016). Area Equipped for Irrigation represents the potential extension of irrigation accounted for by official institutions and governments. The second data set relies on Siebert et al. (2015)’s AEI values between 1950 and 2005, which result from the combination of FAO’s data with national and sub-national surveys, land cover maps and independent remote sensing imagery. The third data set uses values from Thenkabail et al. (2009) on Annualized Irrigated Areas (AIAs) at the end of the last millennium, which represent the sum of the irrigated areas during the two cropping seasons plus continuous year-round irrigation. AIAs were exclusively calculated through remote sensing, Google Earth, and ground control points. The three data sets thus cover all the range from fully officially to fully independently measured irrigated areas. I also paired each country in each data set and time series with its matching total population values, retrieved from the United Nations (UN) Population Division (UN 2015). Appendix S1: Tables S1–S2 summarize the total number of distinct countries per continent included in each data set and time series after merging with the UN population values. Oceania is excluded from the analysis due to its small sample size.

The statistical approach

I assess the relationship between population \( X_{i,t} \) and irrigated area \( Y_{i,t} \) through Standard Major Axis (SMA) regressions for line fitting on log-transformed variables, as log \( y_{i,t} = \log \alpha + \beta \log x_{i,t} \). Unlike Ordinary Least Squares (OLS), which allocates all equation error to Y, SMA splits the equation error between \( Y \) and \( X \) (Warton et al. 2006, Smith 2009). This feature makes OLS a better choice when the aim is to predict \( Y \) from \( X \) or when \( X \) causes \( Y \) and SMA more appropriate when the aim is to summarize the bivariate scatter between \( X \) and \( Y \) or to assess how the proportions between \( X \) and \( Y \) change with a change in size. Due to the presence of outliers (Appendix S1: Fig. S1), the relationship and line of best fit between population and irrigated area is determined as follows: when the sample size is \( n > 50 \), I use the Huber’s M estimator. When \( 5 < n < 50 \), I use the Huber’s M estimator in combination with the fast-and-robust bootstrap (\( B = 10,000 \)) method by Salibián-Barrera et al. (2008), adapted to SMA regression by Taskinen and Warton (2011). The fast-and-robust bootstrap by Taskinen and Warton uses the bias-corrected and accelerated (BC\(_a\)) method to compute confidence intervals (CI) and achieves near optimum coverage probability for small sample sizes. When \( n < 5 \), no regressions were conducted as the fast-and-robust bootstrap method was not stable. All statistical tests are carried out in the R Core Team (2018, version 3.4.4) and the full code and the data sets are available in Data S1.

RESULTS

Table 1 shows Pearson’s \( r \) and Spearman’s \( \rho \) as measures of correlation between irrigated area and population, with the data grouped by continents. The values evidence the existence of a strong relationship between both variables, especially in the case of the Americas (\( r, \rho \geq 0.91 \)), Asia (\( r, \rho \geq 0.88 \)) and Europe (\( r, \rho \geq 0.73 \)). Africa shows a lower

<table>
<thead>
<tr>
<th>Data set</th>
<th>Continent</th>
<th>( n )</th>
<th>( r )</th>
<th>Spearman ( \rho )</th>
<th>( r^2 )</th>
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<tr>
<td>Aquastat</td>
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<td>0.96</td>
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<tr>
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<td>34</td>
<td>0.91</td>
<td>0.91</td>
<td>0.83</td>
</tr>
<tr>
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<td>Asia</td>
<td>42</td>
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<td>Africa</td>
<td>47</td>
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<td>0.61</td>
<td>0.47</td>
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</table>
correlation ($r, p \geq 0.61$). The results suggest that irrigated areas and population are codependent at the national/continental level and that their relationship can be summarized with a line of best fit in log-transformed axes following Eq. 1.

Fig. 1A and B and Table 1 present the results of the SMA regressions. The statistical fit of the model is high for the Americas ($0.83 \leq r^2 \leq 0.92$) and Asia ($0.78 \leq r^2 \leq 0.83$) and lower for Europe ($0.57 \leq r^2 \leq 0.74$) and Africa ($0.47 \leq r^2 \leq 0.53$).

![Results of the Standard Major Axis (SMA) regressions. (A) Lines of best fit. (B) 95% CI for $\beta$. The vertical red dashed line is at $\beta = 1$. (C) Probability of getting $\beta \leq 1$ as a function of the sample size. The dashed black line is at $p = 0.05$. (D) 95% CI for $\beta$ per quantile of population size after pooling all the observations in each data set. The vertical red dashed line is at $\beta = 1$. (E) 95% CI for $\beta$ per quantile of population size with the data split by continents. The vertical red dashed line is at $\beta = 1$. The colors reflect the quantiles as in panel D).
The 95% CI for $b$ show $b > 1$ in all cases, suggesting that, on average, irrigated areas grow disproportionately for a given increase in population.

Aiming at assessing the robustness of this superlinear relationship, I check its consistency across different population cut-offs and time periods. This implies conducting the SMA regressions on fractions of the total sample size, which increases the chances of committing Type II errors in the estimation of $b$ (Hui and Jackson 2007). In order to assess how different sample sizes affect the chances of misidentifying $b$, I rely on a bootstrap approach: for each continent in each data set I randomly draw, without replacement, a sample size $n$, for $n = 5, 6, ..., N$ observations and $N$ the maximum sample size in any time period, and repeat this process 1,000 times. Then, for each sample size $n$ I calculate the 95% CI for $b$ and define beta error as the probability of $b \leq 1$, since I know that $b > 1$ in all continents and data sets (Fig. 1B). The results are summarized in Fig. 1C and Appendix S1: Table S3. The critical sample size $n_c$ needed to reduce Type II errors below 5% varies as a function of data set and continent, with Asia in the Aquastat and the Siebert et al. (2015)’s data sets requiring sample sizes larger than 50.

I first assess the consistency of $b > 1$ across different population cut-offs in order to check the scale-invariant properties of the growth rate between population and irrigated areas. Previous work on cities has shown that, depending on the population cut-off (e.g., $10^4$ individuals, $5 \times 10^4$ individuals), cities might show $\beta \approx 1$ or $\beta \neq 1$ for the same attribute or indicator (Arcaute et al. 2015). In our case, the lack of any standard measure to classify countries as a function of their population size makes the selection of any population cut-off a somewhat arbitrary exercise. To minimize this caveat, I assess whether $b$ shows self-similarity when the 25th, 50th and 75th quantiles of population values are used as cut-off points. The results are presented in Fig. 1D and E and Appendix S1: Fig. S3. When the tests are conducted on the pooled data, all data sets show $b > 1$ in all quantiles. When the data is split by continents, the Americas in the Thenkabail et al.’s (2009a) data set shows all three quantiles with $b_{25} > 1$. This is reasonably a Type II error: the tests were conducted on sample sizes $n$ of 9, 18, and 26 observations, which offer, respectively, 70%, 52%, and 22% chances of yielding $b \leq 1$ (Fig. 1C). A critical sample size $n_c$ of 31 is needed to reduce below 5% the chances of committing a beta error for the Americas in the Thenkabail et al.’s (2009a) data set (Appendix S1: Table S3).

I then assess the robustness of $b > 1$ between 1950 and 2017 by splitting the Aquastat and Siebert et al.’s (2015) data sets in decades. The Thenkabail et al.’s (2009b) data set
included data collected only at the end of the last millennium and therefore was not retained for the time-series analysis. The results are shown in Fig. 2, Appendix S1: Figs. S2 and S4. The superlinear regime between population and irrigated area holds in all continental levels and time series except for Asia between 2008 and 2017, where $\beta \approx 1$. However, compared to 1988–1997 ($n = 53$) and 1998–2007 ($n = 51$), the sample size available for Asia in 2008–2017 ($n = 19$) is much smaller. According to Fig. 1C, the chances of obtaining $\beta \approx 1$ for Asia in the Aquastat data set with $n = 19$ are 78%, a non-negligible Type II error rate. This also questions the reliability of $\beta \approx 1$ for Asia in 2008–2017.

### Discussion and Conclusions

Once the effects of sample size are accounted for, the results suggest the existence of a superlinear growth rate (e.g., with $\beta > 1$) between population and irrigated areas. In elasticity terms, a 1% increase in population is associated, on average, with more than 1% increase in the extension under irrigation. This behavior is robust against variations in the extension of irrigated areas documented by official estimations and independent scholars. It also seems to hold for all continents in all time series under consideration and across different population cut-offs, thus indicating a certain degree of self-similarity. The results contrast with observations by Tilman et al. (2002) and Stewart (2009), who pointed out that irrigated areas have grown slower than population since 1978–1988, e.g., with $\beta < 1$. The sensitivity of the estimation of $\beta$ to sample size stresses the need to control for Type II errors and handle large data sets to prevent confusing a consistent superlinear relationship with a context-dependent, fluctuant growth rate.

The systematic behavior identified in this paper suggests that the extension of irrigation at the national/continental levels might not be critically determined by transient, local particularities (e.g., climate; political, social or economical organizations; food/biofuel demands) or path-dependencies (e.g., historical background, technological development). Instead, it indicates that the complex paths leading to the expansion of irrigated areas can be fairly abridged into a power function of population size. This is especially the case for the Americas, Asia, and Europe, whereas for Africa, the poorer fit with the log-transformed version of Eq. 1 hints at other factors having a higher role in defining irrigated areas. In any case, this paper provides strong support for intensive food-producing systems being governed by the so-called “aggregate effect,” e.g., nonlinear dynamics that emerge from large groups of people and give rise to behaviors that can not be described by the sum of its individual parts (Byrne 1998). This effect has been observed in cities, where larger concentrations of population lead to accelerated socioeconomic processes, e.g., much higher crime and innovation rates (Bettencourt et al. 2007, Bettencourt 2013). Here I suggest that larger groups of people rely on disproportionately larger intensive agrarian systems to feed their swifter pace of life and sustain their increasingly quicker economic growth, which is also based on the exchange of agrarian produces. The influence of such nonlinearities in shaping the extension of irrigation opens up two highly relevant issues for sustainability and policy-making.

1. **The problem of finite time singularity.** Unlike linear ($\beta \approx 1$) and sublinear ($\beta < 1$) growth rates, whose behavior can be respectively described by exponential and sigmoidal curves, superlinear growth rates ($\beta > 1$) become infinitely large at some finite time (Bettencourt et al. 2009). This means that irrigated areas expand unbounded and will eventually reach a point where endless resources, e.g., land and water, will be required to sustain such growth, hence the singularity. Without this infinite supply, irrigated agriculture risks transitioning into an unknown regime and/or collapsing. Following West (2017), the only way to maintain a superlinear growth rate on finite resources is to push the singularity into the future by finding innovative ways to spare such resources: the invention of irrigation technologies (e.g., drip, sprinkler) and chemical fertilizers are conspicuous examples. However, the singularity will be approached again and a new innovation would be required to push it further to the future, with the cycles between innovations and upcoming singularities getting increasingly shorter. The open-ended growth of irrigated areas indicates that we are bound to keep on designing solutions to spare water and land in an ever-increasing pace if we aim at preventing an agrarian regime shift with unpredictable consequences. This is specially alarming considering that irrigated areas scale superlinearly with population and that zero population growth seems unachievable within the next 100 years (FAO 2017).

2. **Irrigated areas might scale with other attributes.** It could be that the nonlinear growth rate of irrigated areas causes nonlinear effects in relevant social-ecological attributes. Previous work on case studies has suggested that the size of irrigated areas does condition to some extent agricultural production, the volume of freshwater withdrawal or environmental impact, among others (Netting 1993, Dillon 2011, Lankford et al. 2016). Yet we do not know whether any of these attributes increase linearly, sublinearly, or superlinearly for every increase in the area under irrigation. Such knowledge is critical to better appraise the risks and benefits derived from extending irrigated areas, for instance: if freshwater withdrawal scales sublinearly, e.g., $\beta < 1$ (or superlinearly, e.g., $\beta > 1$) with irrigated areas, it will (or will not) pay off, on average and ceteris paribus, to promote larger irrigated areas, as every increase in the area under irrigation will lead to marginal (or accelerated) increases in the volume of freshwater withdrawn. Identifying such patterns via scaling will allow us to know which nonlinearities are worth benefiting from and which should be avoided, therefore grounding our policies on irrigated agriculture in more solid scientific foundations.

Although the superlinear growth rate identified in this paper is documented from 1950 onward, it could extend further back into the past. Scaling properties attested in current urban settings have also been documented in prehistorical and medieval contexts (Ortman et al. 2014, 2015, Cesaretti et al. 2016), suggesting that scaling relationships might be impermeable to historical contingencies. Checking this out in the case of irrigated areas requires a systematic collection of diachronic data on the changes in size undergone by
ancient irrigated areas and related settlement clusters. Although much data on the latter is retrieved by archaeologists, the former tends to be overlooked despite the existence of a consolidated methodology developed to that aim (Hydraulic Archaeology, see Kirchner 2009; Puy 2014). Assessing the relationship between irrigated areas and population in historical contexts will help strengthening or nuances the validity of the superlinear growth rate unfolding here, with strong implications for our understanding of the coevolution between humans and agricultural systems and the social-ecological dynamics of ancient hydraulic societies.

It is also worth stressing that other agrarian systems might scale differently with population. Dryland agriculture, for instance, is often linked to abundant land and sparse groups of individuals. Farmers might switch to more intensive agrarian systems following a significant increase in land or population pressure (Netting 1993). This suggests that, unlike irrigated agriculture, dryland agriculture is likely to grow either linearly or marginally for any given increase in population (e.g., with $\beta \leq 1$), a growth rate whose implications for environmental sustainability are not as deleterious. Determining whether this is the case might help us better understand processes of agrarian change and break through the Boserup debate on the paths toward the adoption of intensive agrarian systems (Boserup 1965, 1981). Overall, the application of scaling to study food-producing systems will provide us with an enhanced perspective on how they relate to population, a more nuanced understanding of how size conditions their behavior, as well as better instruments to secure human welfare without compromising environmental integrity.

**Acknowledgments**

This work was supported by the European Commission (Marie Curie IEF, grant number 623098). I thank Sarah Taskinen (University of Jyväskylä, Finland) for facilitating the fast-and-robust bootstrap method in the R environment. All mistakes and shortcomings are my own.

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**Supporting Information**

Additional supporting information may be found online at: http://onlinelibrary.wiley.com/doi/10.1002/eap.1743/full

**Data Availability**

The data sets and code can be found in Data S1.